

Computational Issues in Optimal Tuning and Placement of Passive Dampers

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Abstract

The effectiveness of viscous elements in introducing damping in a structure is a function of several variables, including their number, their location in the structure, and their physical properties. In this paper, the optimal damper placement and tuning problem is posed to optimize these variables. Both discrete and continuous optimization problems are formulated and solved, corresponding, respectively, to the problems of placement of passive elements and to the tuning of their parameters. The paper particularly emphasizes the critical computational issues resulting from the optimization formulations. Numerical results involving a lightly damped testbed structure are presented.

1. Introduction

A problem of considerable importance in the development of technology for future space structures is the analysis and optimization of passive elements placed in these structures. Passive damping introduced by these devices is an effective mechanism for reducing peak responses in the vicinity of resonant frequencies for lightly damped systems. This not only enhances the stability of the open-loop system, but also allows for the implementation of more aggressive control strategies to achieve greater performance. This philosophy is being pursued on a series of Control Structure Interaction (CSI) testbeds at the Jet Propulsion Laboratory.

The effectiveness of viscous elements in introducing damping is a function of several variables, including their number, their location in the structure, and their physical parameters, namely damping and stiffness coefficients. This paper is concerned with the optimal placement and tuning problem for the passive viscous dampers with emphasis on its computational aspects.

Two qualitatively different optimization problems arise in this context: a combinatorial optimization problem which determines the placement of elements, and a mathematical programming problem which optimizes (tunes) the damper parameters. In our approach, a simulated annealing strategy [4] is used for the combinatorial optimization problem, while a sequential quadratic programming algorithm (SQP) [2] is applied to the damper parameter optimization problem. One of the most important ingredients in any optimization problem is the cost functional evaluation, regardless of the performance metric that is used. This is particularly true for the optimal damper placement and tuning problem due to the

complexity of the system. The performance metric chosen here is the \mathcal{H}_2 -norm of selected transfer functions of interest. An excellent candidate is the transfer matrix between external disturbance inputs and the controlled outputs.

It is well known that the computation of the \mathcal{H}_2 -norm requires solving a Lyapunov equation. However, due to the high-dimensionality of the system model, it is unrealistic to use the full-order model in any computation. A reduced-order model must be generated to make the computation involved more manageable. The Ritz reduction method that has been studied in [1] is employed to reduce the numerical bottleneck created by solving large systems of this type.

The paper is organized as follows. Section 2 presents the dynamic model of a viscously damped structure. The general optimal damper placement and tuning problem is formulated in Section 3 with a review on the computation of the \mathcal{H}_2 -norm of the particular transfer matrix which is chosen to be our performance metric. Section 4 addresses the computation issues involved in our optimization problem. In particular, the Ritz reduction method will be described in detail. A number of numerical examples involving the JPL testbed structure are presented in Section 5. Finally, concluding remarks on future work are given in Section 6.

2. Dynamic Modeling for Viscously Damped Structures

Throughout this paper, it is assumed that the dynamics of the undamped structures can be described by a linear, second-order matrix differential equation of the form:

$$M\ddot{z} + Kz = B_d d . \quad (1)$$

Here z denotes the n -dimensional vector of generalized coordinates, d is an l -dimensional external forcing input vector, M is the $n \times n$ symmetric, positive definite mass matrix, K is the $n \times n$ symmetric, positive definite stiffness matrix, and B_d is the $n \times l$ forcing input influence matrix.

Assume that a discrete passive damper is placed between two nodal points in the structure, replacing the original structural element. The passive damper is modelled as a device that applies a force at the nodal points with equal magnitude but in opposite directions and proportional to the relative displacement and velocity between the nodal points.

The dynamic structural model incorporating the damper actuator force, u , is written as

$$M\ddot{z} + Kz = bu + B_d d \quad (2)$$

where the vector b represents the influence vector associated with u . The force u generated by the damper is modelled as a constant linear combination of collocated position and

velocity feedback so that

$$u = -(k_p y_p + k_v y_v) \quad (3)$$

with $y_p = b^T z$ and $y_v = b^T \dot{z}$ where y_p and y_v denote the position and velocity “measurements,” respectively, and k_v denotes the damping rate, which is always taken as a nonnegative quantity to ensure stability. The parameter k_p is only required to be greater than or equal to the value $-k_e$, where k_e denotes the stiffness of the structural element that has been replaced by the damper. When $-k_e \leq k_p < 0$, the structure is softened, while $k_p > 0$ causes the structure to be stiffened.

Hence, the dynamic structural model with the inclusion of a passive damper can be represented as

$$M\ddot{z} + (K + k_p b b^T)z = b(-k_p b^T z - k_v b^T \dot{z}) + B_d d, \quad (4)$$

or

$$M\ddot{z} + (k_v b b^T)\dot{z} + (K + k_p b b^T)z = B_d d. \quad (5)$$

A more general model including multiple passive dampers can be written as

$$M\ddot{z} + \left(\sum_{i=1}^{n_p} k_{v,i} b_i b_i^T \right) \dot{z} + \left(K + \sum_{i=1}^{n_p} k_{p,i} b_i b_i^T \right) z = B_d d \quad (6)$$

where n_p is the number of passive dampers in the structure.

3. Optimal Placement and Tuning Problem for Passive Dampers

The general optimal placement/tuning problem of passive dampers can be posed as

$$\min_{K_p \in \mathcal{K}_p, K_v \in \mathcal{K}_v} \min_{B_p \in \mathcal{B}_p} \mathcal{J}_{cost}(B_p, K_p, K_v) \quad (7)$$

where

- $\mathcal{J}_{cost}(B_p, K_p, K_v)$ is defined as the performance metric for the optimization with a given damper configuration of locations corresponding to B_p and the corresponding stiffness and damping rate K_p and K_v
- $\mathcal{B}_p \stackrel{\Delta}{=} \{(b_{i_1}, b_{i_2}, \dots, b_{i_{n_p}}) : i_1, i_2, \dots, i_{n_p} \in \mathcal{N}_p, i_\alpha \neq i_\beta, \forall \alpha, \beta = 1, 2, \dots, n_p (\alpha \neq \beta)\}$
(b_{i_α} is the influence vector corresponding to the i_α^{th} location).
- $\mathcal{K}_p \stackrel{\Delta}{=} \{(k_{p,i_1}, k_{p,i_2}, \dots, k_{p,i_{n_p}}) : i_1, i_2, \dots, i_{n_p} \in \mathcal{N}_p, i_\alpha \neq i_\beta, \forall \alpha, \beta = 1, 2, \dots, n_p (\alpha \neq \beta)\}$
($k_{p,j}$ is the stiffness correction corresponding to the damper at j^{th} location, and

$$k_{s_{min}} \leq k_{p,j} + k_{e,j} \leq k_{s_{max}}$$

where $k_{e,j}$ is the element stiffness of the undamped structure at j^{th} location, $k_{s_{min}}$ and $k_{s_{max}}$ are the lower and upper bound of the damper stiffness).

- $\mathcal{K}_v \triangleq \{(k_{v_{i_1}}, k_{v_{i_2}}, \dots, k_{v_{i_{n_p}}}) : i_1, i_2, \dots, i_{n_p} \in \mathcal{N}_p, i_\alpha \neq i_\beta, \forall \alpha, \beta = 1, 2, \dots, n_p (\alpha \neq \beta)\}$
 $(k_{v_j}$ is the damping rate corresponding to the damper at j^{th} location, and

$$0 \leq k_{v_j} \leq k_{v_{max}}$$

where $k_{v_{max}}$ is the highest possible damping rate for the passive damper).

- \mathcal{N}_p is defined as the set of all candidate locations for placement.

It is clear that the above optimization problem is a joint “continuous+discrete” optimization problem. The selection of locations (B_p) for placement is a “discrete” combinatorial optimization problem while the selection of values for K_p and K_v (tuning) is a continuous mathematical programming problem.

Two types of performance metrics are typically considered. The first one is the structural modal damping for selected modes. The computation involved is to solve for the eigenvalues of the “A”-matrix obtained from writing (6) in first-order form for a given damper configuration with corresponding damper stiffness and damping coefficients. The second type of criterion requires both the external disturbance input vector and the controlled output vector to be specified. As discussed in [6], a meaningful and numerically tractable criterion for the associated optimization problem is to minimize the \mathcal{H}_2 -norm of the transfer function from d to y_o . In addition, a weighting function $W_d(s)$ can be used to model the spectral property of d and a weighting function $W_p(s)$ can be used to improve the performance of y_o over a certain frequency range. In this case, the cost functional is simply

$$\mathcal{J}_{cost} = \|W_p(s)G_p(s; B_p, K_p, K_v)W_d(s)\|_2 \quad (8)$$

where $G_p(s; B_p, K_p, K_v)$ is defined as the transfer matrix from the d to y_o with a given damper configuration of locations corresponding to B_p and with the corresponding stiffness and damping coefficients, K_p and K_v . For a given damper configuration (B_p, K_p, K_v) and the weighting functions $(W_p(s), W_d(s))$, the \mathcal{H}_2 -norm can be computed through the solution of a specific Lyapunov equation.

Define

$$T(s) = W_p(s)G_p(s; B_p, K_p, K_v)W_d(s)$$

and assume that $T(s)$ has the state-space realization (A, B, C) where the matrix A is asymptotically stable. Then the corresponding \mathcal{H}_2 -norm of $T(s)$ is simply

$$\|T(s)\|_2 = [\text{trace}(CPC^T)]^{1/2} = [\text{trace}(B^TQB)]^{1/2}$$

where P and Q are the positive semi-definite solutions of the following two Lyapunov equations:

$$AP + PA^T + BB^T = 0 \quad (9)$$

and

$$A^T Q + QA + C^T C = 0 , \quad (10)$$

respectively [3].

4. Computational Issues and Model Reduction

As discussed in the previous section, the damper placement and tuning problem includes solving a nonlinear mathematical programming problem for tuning, and a combinatorial optimization problem for placement.

In particular, the combinatorial optimization problem is known to be difficult due to the fact that the potential number of candidate locations for placement (N_p) will be large in large space flexible structures. However, relatively few passive devices (n_p) will be available. In general, $N_p >> n_p$, and the total number of combinations, $\frac{N_p!}{n_p!(N_p-n_p)!}$, is usually very large. Therefore, it is impractical, if not completely impossible, to try the exhaustive search.

In our approach, a sequential quadratic programming algorithm (SQP) [2] is applied to the damper parameter tuning problem while a simulated annealing strategy [4] is used for the combinatorial optimization problem. The question of developing a hybrid approach for combining these strategies into a single approach will not be dealt with here and is one of our future research topics.

Our current strategy is to solve each of these problems individually. One approach is to solve the damper parameter tuning problem for each candidate location first. These parameters will then be used to evaluate the cost functional in the simulated annealing process.

Another approach is to use a "pruning" process after each of the candidate locations is "tuned." This pruning process is simply to choose the top N_p' candidate locations according to the ranking of their respective optimized cost functional where $N_p >> N_p' > n_p$. An exhaustive combinatorial search is then conducted throughout this subset to find the "optimal" combination of elements which yields the smallest \mathcal{H}_2 -norm cost. This *ad hoc* pruning approach has been demonstrated to be quite useful. However, it is difficult to make a general statement regarding the solutions of these sub-optimal approaches as compared to the optimal ones.

As stated in the Introduction, one of the most important ingredients in any optimization problem is the cost functional evaluation. This is particularly true for the optimal damper placement and tuning problem due to the complexity of the system. The performance metric chosen here is the \mathcal{H}_2 -norm of selected transfer functions of interest.

The procedure to compute the \mathcal{H}_2 -norm of a stable transfer matrix has been given in Section 3 and requires solving a Lyapunov equation. However, it is impractical, if not impossible, to use the full-order model in the computation of the \mathcal{H}_2 -norm since the order of the model,

$2 \times n$, is typically very large. Hence, a high-fidelity, low-order, reduced model must be used to perform the required computation efficiently.

The Ritz reduction method that has been studied in [1] is employed to reduce the numerical bottleneck created by solving large systems of this type. Details of this model-reduction method will be described in the rest of this section.

The Ritz Reduction Method

To solve the optimization problem posed in the previous section, it is impractical, if not impossible, to use the full-order model in the optimization process since the order of the model, $2 \times n$, is typically very large. Hence, a high-fidelity, low-order, reduced model must be used to perform the required computation efficiently.

The model-reduction method considered here is a second order reduction technique based on reducing the number of generalized coordinates of the system via a transformation of the form $z = \mathcal{P}q$, where $q \in R^N$ with $N < n$. Applying the transformation \mathcal{P} to (6) results in the reduced-order model

$$(\mathcal{P}^T M \mathcal{P})\ddot{q} + \left[\sum_{i=1}^{n_p} k_{v_i} (\mathcal{P}^T b_i)(\mathcal{P}^T b_i)^T \right] \dot{q} + \left[(\mathcal{P}^T K \mathcal{P}) + \sum_{i=1}^{n_p} k_{p_i} (\mathcal{P}^T b_i)(\mathcal{P}^T b_i)^T \right] q = (\mathcal{P}^T B_d) d . \quad (11)$$

The transformation matrix, \mathcal{P} , consists of the first m ($m \ll n$) eigenvectors corresponding to the first m eigenvalues, $\{\omega_1, \omega_2, \dots, \omega_m\}$, and an additional Ritz vector to account for the static correction for each of the forcing inputs. This method will be referred to as the "Ritz reduction method." A detailed discussion on this subject can be found in [1].

Suppose that the lowest m eigenvalues and their corresponding eigenvectors are known and Φ_m is defined as the $n \times m$ matrix consisting of the m eigenvectors corresponding to $\{\omega_1, \omega_2, \dots, \omega_m\}$. Then the desired Ritz vector corresponding to b_i (i^{th} damper) is simply the solution to the following linear equation:

$$K \psi_i = b_i .$$

It is desirable for the transformation matrix to preserve M -orthonormality. Therefore, ψ_i needs to be M -orthonormalized. This is done easily by first

1. making ψ_i M -orthogonalized to Φ_m

$$\tilde{\psi}_i = \psi_i - \Phi_m (\Phi_m^T M \psi_i) \quad (12)$$

and then

2. making $\tilde{\psi}_i$ M -normalized, i.e.,

$$\phi_r = (\tilde{\psi}_i^T M \tilde{\psi}_i)^{-1/2} \tilde{\psi}_i . \quad (13)$$

Similarly, the desired Ritz vector corresponding to b_{d_j} (j^{th} external disturbance input) can be computed using the same procedure.

Note that for each of the forcing inputs, one Ritz vector needs to be computed. The forcing inputs could be either the force inputs corresponding to the dampers or external disturbance inputs. Let ϕ_i^r denote the M -orthonormalized Ritz vector corresponding to the i^{th} influencing input vector, b_i , and $\phi_{d_j}^r$ denote the M -orthonormalized Ritz vector corresponding to the j^{th} external disturbance influencing input vector, b_{d_j} . Note that each of the corresponding Ritz vectors is M -orthonormalized to Φ_m ; however, the $(n_p + l)$ Ritz vectors may not be M -orthogonal among themselves. An additional M -orthogonalized step is required. Define

$$\tilde{\Phi}_{ritz} = [\phi_1^r \ \phi_2^r \ \dots \ \phi_{n_p}^r \ \phi_{d_1}^r \ \phi_{d_2}^r \ \dots \ \phi_{d_l}^r]$$

and form

$$M_{ritz} = \tilde{\Phi}_{ritz}^T M \tilde{\Phi}_{ritz}, \text{ and } K_{ritz} = \tilde{\Phi}_{ritz}^T K \tilde{\Phi}_{ritz}$$

to find $\hat{\Phi}_{ritz}$ such that $\hat{\Phi}_{ritz}$ is M_{ritz} -orthonormalized, i.e.,

$$\hat{\Phi}_{ritz}^T M_{ritz} \hat{\Phi}_{ritz} = I_{(n_p+l) \times (n_p+l)}, \text{ and } \hat{\Phi}_{ritz}^T K_{ritz} \hat{\Phi}_{ritz} = \Omega_{ritz}^2$$

where $\Omega_{ritz} = \text{diag}[\omega_{r_1} \ \omega_{r_2} \ \dots \ \omega_{r_{n_p+l}}]$.

Define $\Phi_{ritz} = \tilde{\Phi}_{ritz} * \hat{\Phi}_{ritz}$, then the M -orthonormal transformation matrix \mathcal{P} is

$$\mathcal{P} = [\Phi_m \ \Phi_{ritz}]$$

and Eq. (11) is equivalent to

$$I_{N \times N} \ddot{q} + \left(\sum_{i=1}^{n_p} k_{v_i} (\mathcal{P}^T b_i) (\mathcal{P}^T b_i)^T \right) \dot{q} + \left[\Omega_N^2 + \sum_{i=1}^{n_p} k_{p_i} (\mathcal{P}^T b_i) (\mathcal{P}^T b_i)^T \right] q = (\mathcal{P}^T B_d) d \quad (14)$$

where $N = m + n_p + l$ is the order of the reduced model, and $\Omega_N = \text{diag}[\Omega_m \ \Omega_{ritz}]$.

The reduced-order model in Eq. (10) can also be rewritten in the state-space representation as

$$\dot{x} = \begin{bmatrix} 0_{N \times N} & I_{N \times N} \\ -\Omega_N^2 - \sum_{i=1}^{n_p} k_{p_i} (\mathcal{P}^T b_i) (\mathcal{P}^T b_i)^T & -\sum_{i=1}^{n_p} k_{v_i} (\mathcal{P}^T b_i) (\mathcal{P}^T b_i)^T \end{bmatrix} \dot{x} + \begin{bmatrix} 0_{N \times m} \\ \mathcal{P}^T B_d \end{bmatrix} d \quad (15)$$

where $x = \begin{bmatrix} q \\ \dot{q} \end{bmatrix}$ is the state vector.

5. Numerical Examples

A detailed description of the JPL testbed can be found in [5] (see Figure 1). Briefly, the system is modeled with 249 degrees of freedom and contains 186 candidate locations to insert passive damping elements.

Because the accuracy of the cost functional evaluation methods is of paramount importance in the optimization process, Table 1 contains a comparison of eigenvalue approximations using the full-order model, the Ritz reduced model, and a modally reduced model. The second column in part (a) of the table contains the eigenvalues of the undamped nominal system. All of the other values correspond to the damped system with three viscous dampers placed at the locations 132, 140, and 142. It is assumed that the three dampers have the same damping and stiffness coefficients: 320 $\text{lbs} - \text{sec/in}$ and 8,000 lbs/in respectively.

The conclusion here is that the Ritz reduction method yields high-precision estimates with enormous reduction in computation. In this example, instead of solving a 498×498 eigenvalue problem, the results can be obtained by solving a 30×30 eigenvalue problem which results from the Ritz reduction method. However, the modally reduced model produces inaccurate results. What is of equal significance is that not only does the modally reduced model produce inaccurate results, it also leads to inaccurate trends for choosing damper parameters. Figure 2 contains damping predictions of the second system mode as a function of the damper viscous parameter coefficient. Note that the full and Ritz reduced models lead to an optimal coefficient of approximately $500 \text{ lbs} - \text{sec/in}$, while the modally reduced model leads to a significantly larger value that is far from optimal. The Ritz reduction method also leads to very accurate approximation to the \mathcal{H}_2 -norm, with 6 digits of accuracy.

Table 2 contains the eigenvalues of the damped system where the three dampers are placed at the locations 6, 19 and 91. The three locations are the simulated annealing solution to the optimal damper placement problem. The performance metric is the \mathcal{H}_2 -norm of the transfer function from an input disturbance located at grid point 412 between the third and fourth bays of the structure, to the outputs consisting of all of the nodal displacements directly beneath the trolley (see Fig. 1). The disturbance was generated as the output of a 6th-order low-pass filter with a bandwidth of 25 Hz. This weighting function is chosen to reflect the objective of disturbance reduction in the frequency range below 25 Hz. A representative comparison of the undamped and damped frequency responses is given in Figure 3.

6. Concluding Remarks

The use of strategically placed and tuned passive elements in future large space structures will play a significant role in their design and development. The ability to analyze, predict, and ultimately optimize system performance with respect to these passive devices is critical for the application of this damper placement technology.

A comprehensive overview of the optimal damper placement and tuning problem was presented in this paper. Approaches and computational aspects of the associated optimization problems were discussed. The results of the paper indicate that significant levels of damping can be introduced into these structures in a very systematic and tailored fashion.

Although reasonably good results have been demonstrated using the approach presented here, the combined discrete plus continuous optimization problem was essentially solved for each individually. This is the major drawback of our current approach. Our future work will concentrate on the development of a hybrid approach to jointly solve the two qualitatively distinct optimization problems.

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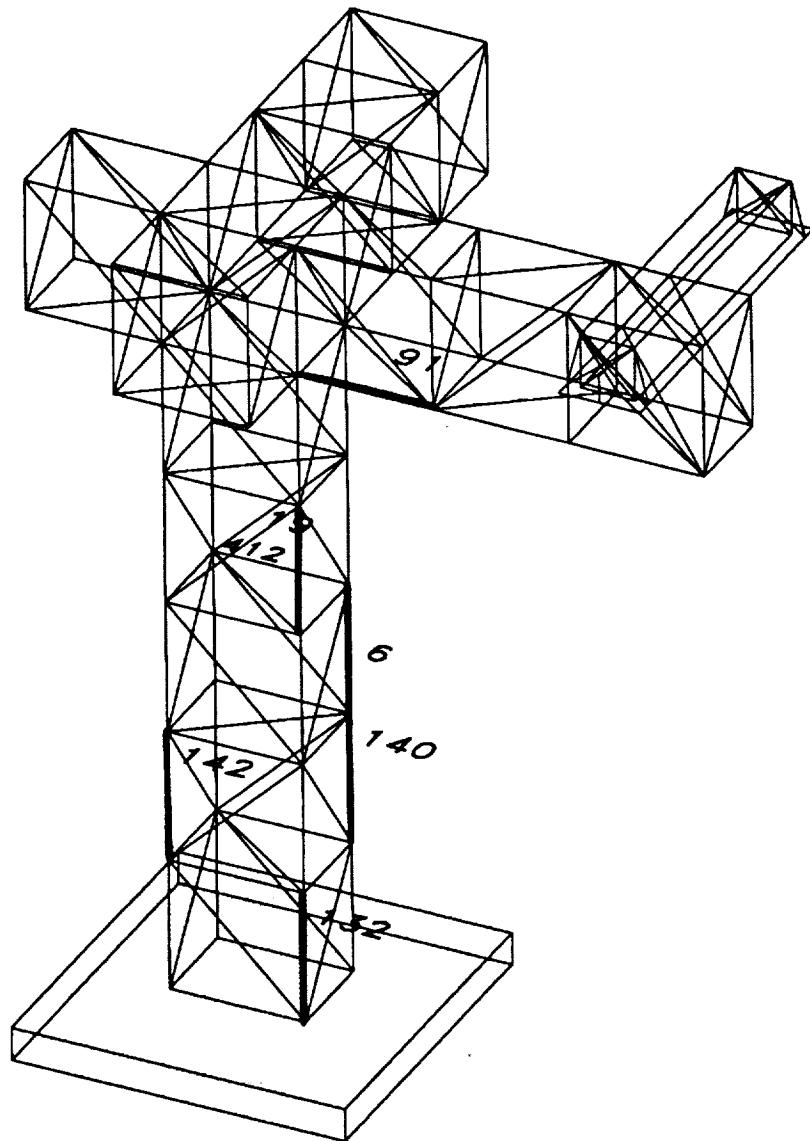


Figure 1. JPL CSI Phase B Testbed

Mode	Undamped System	Damped System		
		249 Modes (Full order)	12 Modes plus 3 Ritz vectors	15 Modes (Truncation)
1	0.7427	0.7420	0.7420	0.7425
2	5.4263	5.2940	5.2940	5.3262
3	7.4565	7.0376	7.0376	6.9540
4	11.6777	10.4862	10.4862	10.4493
5	17.4248	17.4386	17.4386	17.3444
6	20.8423	20.8236	20.8236	20.7055
7	31.1387	31.2231	31.2231	31.0481

(a) Frequency (in Hertz)

Mode	Damped System		
	249 Modes (Full order)	12 Modes plus 3 Ritz vectors	15 Modes (Truncation)
1	0.0179	0.0179	0.0012
2	4.5744	4.5744	0.6125
3	25.5358	25.5357	2.3228
4	32.6380	32.6379	5.5664
5	0.9033	0.9034	0.4066
6	1.3197	1.3197	0.5709
7	0.5013	0.5016	0.5031

(b) Damping (in %)

Table 1. Undamped and Damped Eigenvalues
(Damper Locations: 132, 140, and 142)

Mode	Frequency (Hz)	Damping (%)
1	0.7414	0.0245
2	5.0393	6.8905
3	7.1748	10.0192
4	11.4717	3.7751
5	17.5924	3.2823
6	20.9413	2.0084
7	31.1573	0.0788

Table 2. Eigenvalues of the Damped System with \mathcal{H}_2 -Optimized Damper Locations at 6, 19, and 91.

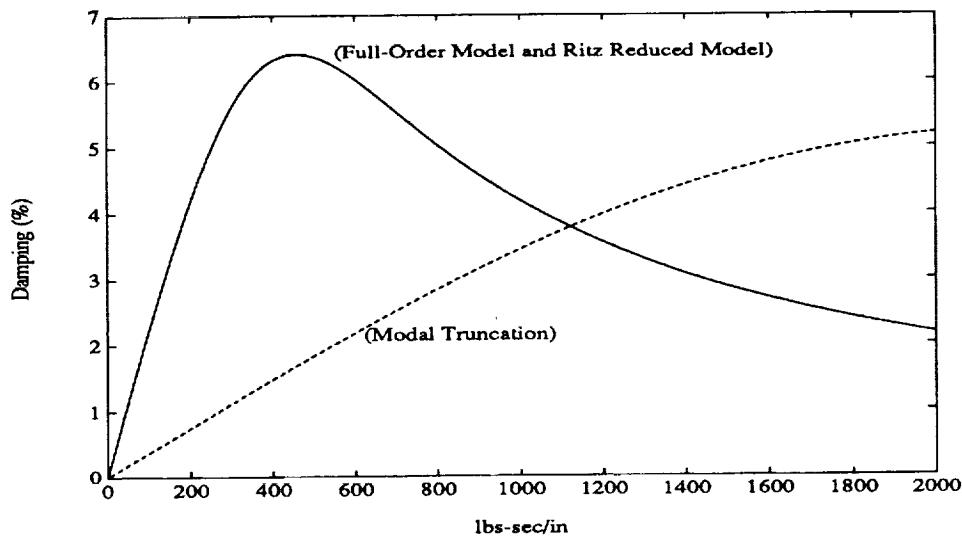


Figure 2. Damping Prediction by Reduction Methods

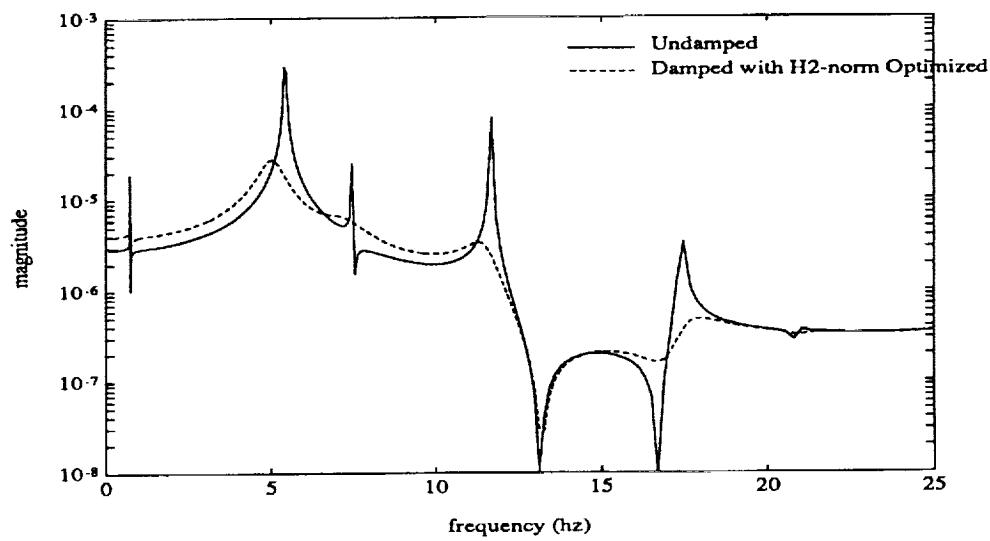


Figure 3. Disturbance Frequency Responses of Undamped and Damped Systems